

Time 3 Hrs

M.M.100

**General Instructions:**

- (i) All questions are Compulsory.  
(ii) This question paper consists of 29 questions divided into three Sections-A, B, C. Section-A, comprises of 10 questions of one mark each, Section-B comprises of 12 questions of four marks each, Section-C comprises of 07 questions of six marks each,  
(iii) All questions in section-A are to be answered in one word, one sentence or as per requirement of the question.  
(iv) There is no overall choice. However internal choice has been given in 04 questions of four marks and two questions of six marks each. You have to attempt only one of alternate in such questions.  
(v) Use of calculator is not permitted. You may ask for log tables if required.

**Section-A**

Q.1. Find the domain of the function-

$$f(x) = \frac{1}{1-x^2}$$

Q.2. Find the value of  $\sec^2(\tan^{-1} 2) + \operatorname{cosec}^2(\cot^{-1} 3)$ .Q.3. If  $\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$  find a and b

Q.4. What type of matrix is the matrix A

$$A = \begin{bmatrix} 0 & 3 & 4 \\ -3 & 0 & 5 \\ -4 & -5 & 0 \end{bmatrix}$$

Q.5. Without expanding evaluate the determinant  $\begin{vmatrix} 8 & 1 & 2 \\ 12 & -5 & 3 \\ 16 & 2 & 4 \end{vmatrix}$ Q.6. Find the derivative of  $\tan x$  w.r.t.  $\sin x$ .

Q.7. Find the slope of the normal to the curve.

$$y = 5x^2 + 7 \text{ at } x = 2.$$

Q.8. Let a be the given vector whose initial and terminal points are P(2, 3) and Q(3, 7) respectively. Find the components of a and magnitude of a.

Q.9. Find the direction cosines of the vector  $-2i + 3j + 5k$ .

Q.10. Two lines have direction ratios 3, 2, -1 and -2, -1, -8 respectively. Find the angle between them.

**Section-B**Q.11. Prove that the relation R on set Z of all integers defined by  $(a, b) \in R \Leftrightarrow (a - b)$  is divisible by 5 is an equivalence relation on Z.

OR

Check whether  $F: N \rightarrow N$  given by

$$F(x) = \begin{cases} x+1, & x \text{ is odd} \\ x-1, & x \text{ is even} \end{cases} \text{ is both one - one and into}$$

Q.12. Prove that  $\sin^{-1}\left(\frac{5}{13}\right) + \sin^{-1}\left(\frac{7}{25}\right) = \cos^{-1}\left(\frac{253}{325}\right)$ 

Q.13. Using properties of determinants, prove that

$$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$$

(2)

Q.14. Find the derivative of  $\left(\frac{a \cos x - b \sin x}{b \cos x + a \sin x}\right)$  w.r.t.x.

Q.15. Examine the continuity of  $f$  where  $f$  is defined by

$$f(x) = \begin{cases} \sin x - \cos x, & \text{if } x \neq 0 \\ -1 & \text{if } x = 0 \end{cases}$$

Q.16. A Particle moves along the curve  $6y = x^3 + 2$ . Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate.

Q.17. Evaluate:  $\int \frac{dx}{2 - 3 \cos 2x}$

OR

Evaluate:  $\int (x-2) \sqrt{\frac{x+3}{x-3}} dx$

Q.18. Solve the differential equation

$$x \frac{dy}{dx} - y = (x-1)e^x.$$

Q.19. Show that the family of curves for which the slope of the tangent at any point  $(x, y)$  on it is  $\frac{x^2+y^2}{2xy}$  is given by  $x^2-y^2=cx$ .

OR

Solve the differential equation  $(x-y) \frac{dy}{dx} = x+2y$

Q.20. If  $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$ ,  $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$  and  $\vec{c} = 3\hat{i} + \hat{j}$  are such that vector  $\vec{a} + \lambda\vec{b}$  is perpendicular to  $\vec{c}$ , find the value of  $\lambda$ .

Q.21. Find the coordinates of the foot of the perpendicular drawn from the origin to the plane.

$$2x - 3y + 4z - 6 = 0$$

OR

Find the image of the point  $(1, 6, 3)$  in the line  $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

Q.22. Two balls are drawn at random from a bag containing 3 white, 3 red, 4 green and 4 black balls one by one without replacement. Find the probability that both the balls are of different colours.

### Section-C

Q.23. Using matrix method, solve the system-

$$2x + 3y + 3z = 5, \quad x - 2y + z = -4, \quad 3x - y - 2z = 3$$

Q.24. Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.

OR

Show that the semi-vertical angle of a right circular cone of given surface area and maximum volume is

$$\sin^{-1}\left(\frac{1}{3}\right)$$

Q.25. Evaluate:  $\int_0^{\pi/2} \log \sin x \, dx.$

Q.26. Find the area of the region  $\{(x, y) : 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, 0 \leq x \leq 2\}.$

Q.27. Show that the plane whose vector equation is

$\mathbf{r} \cdot (\hat{i} + 2\hat{j} - \hat{k}) = 1$  and line whose vector equation is  $\hat{r} = -\hat{i} + \hat{j} + \hat{k} + \lambda(2\hat{i} + \hat{j} + 4\hat{k})$  are parallel. Also find the distance between them.

Q.28. There are two factories located at P and Q. From these certain commodity is delivered to each to three depots A, B and C. The weekly requirements of the depots respectively are 5, 5 and 4 units of the commodity, while the production capacities of factories P and Q are respectively 8 and 6 units. The cost of transportation per unit is given below:

		To $\rightarrow$		
		Cost (in Rs)		
From $\downarrow$		A	B	C
		P	16	10
	Q	10	12	10

- (i) How many units should be transported from each factory to each depot, in order that transportation cost is minimum? Solve it graphically.
- (ii) If you find a purse containing an amount of rupees in the play ground, what will you do :-
- Put it with you.
  - Submit it in lost - found deptt.
  - Spend the amount in friends.
  - None of these

29. In a factory machine A produces 30% of the total output, machine B produces 25% and machine C, the rest. The defective output of A, B, C are respectively 1%, 1.2% and 2%. All the three machines working together, produce 10000 items in a day. An item is drawn at random from a day's output and found to be defective. Find the probability that it was produced by machine B.

OR

If a fair coin is tossed 10 times, find the probability of (i) exactly six heads (ii) atleast six heads (iii) almost six heads

(iv) Suppose your friend is cheating the answers in an examination. What will be your response then-  
a. say to stop this b. encourage him/her c. tell the invigilator d. ignore this

**Submitted by-**

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